

Field-controlled conical intersections in the vortex lattice of quasi 2D pure strongly type-II superconductors at high magnetic fields

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It is shown that the Dirac fermion structures created in the middle of the Landau bands in the vortex-lattice state of a pure 2D strongly type-II superconductor at half-integer filling factors can be effectively controlled by the external magnetic field. The resulting field-induced modulation of the magneto-oscillations is shown to arise from Fermi-surface resonance scattering in the vortex core regions. Possible observation of the predicted effect in a quasi 2D organic superconductor is discussed.

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In a pure strongly type-II superconductor under a uniform magnetic field the quasi particle spectrum is gapless in a broad field range below the upper critical field H_{c2} [1],[2],[3]. In this field range scattering of quasi particles by the vortex lattice interferes with the Landau quantization of the electron motion perpendicular to the magnetic field to form magnetic (Landau) Bloch's bands. The physical picture is of an extended Bloch state which breaks magnetically into localized cyclotron orbits [3]. In pure 2D, or quasi 2D superconductors, such as e.g. the organic charge transfer salt $\kappa - (ET)_2 Cu(SCN)_2$ [4], under a magnetic field perpendicular to the easy conducting planes, the underlying normal electron spectrum is fully quantized and the effect of the vortex lattice is very pronounced. Furthermore, due to the suppressed energy dispersion along the magnetic field direction and the particle-hole symmetry inherent to the superconducting (SC) state, the quasi particle spectrum exhibits peculiar features that are missing in the 3D case. For example, at discrete magnetic field values where the chemical potential is located in the middle of a Landau band, so that the underlying normal state spectrum satisfies particle-hole symmetry, the calculated quasi-particle density of states (DOS) has a Dirac Fermion structure [5], which reflects topological singularities at the vortex lattice cores.

In the present paper we reveal a physical mechanism which controls the Dirac fermion structures created in the magnetic Brillouin zone (BZ) of the vortex lattice of 2D strongly type-II superconductors at high magnetic fields, and discuss possible experimental probes of their appearance. The ability to create and control Dirac fermions just by varying an external parameter (magnetic field in our case) is of great importance for future technological applications (see, e.g. [6],[7]). For the present analysis we consider a model of a 2D electron system under a perpendicular uniform magnetic field $\mathbf{H} = (0, 0, H)$, neglecting, for the sake of simplicity, the Zeeman spin splitting and assuming a singlet, s -wave electron pairing. It should be noted that in a 2D or quasi 2D electron system the condition of zero spin-splitting can be realized by tilting the magnetic field direction with respect to the easy

conducting planes [4, 8].

The corresponding equations for the quasi particle states in the mean-field approximation are the well known Bogoliubov de Gennes (BdG) equations in the Landau orbitals representation [1, 9]:

$$\begin{aligned} \sum_{n'} \Delta_{n,n'}(\mathbf{k}) v_{n'}(\mathbf{k}) &= (\varepsilon - \xi_n) u_n(\mathbf{k}), \\ \sum_{n'} \Delta_{n',n}^*(\mathbf{k}) u_{n'}(\mathbf{k}) &= (\varepsilon + \xi_n) v_n(\mathbf{k}), \end{aligned} \quad (1)$$

where the single-electron energy measured relative to the chemical potential μ is given by $\xi_n = \hbar\omega_c (n - n_F)$, $n = 0, 1, 2, \dots$, $n_F = \mu/\hbar\omega_c - 1/2$, and $\omega_c = eH/m^*c$ is the electronic cyclotron frequency. The matrix, $\Delta_{n,n'}(\mathbf{k})$, is diagonal in the magnetic Brillouin zone, but non-diagonal in the Landau-level (LL) indices n, n' . The pair potential $\Delta(\mathbf{r})$ should, in principle, be determined self consistently with the eigenfunctions $v_n(\mathbf{k}), u_n(\mathbf{k})$ [3]. It will be very helpful to avoid the complexity involved in a fully self-consistent approach by assuming $\Delta(\mathbf{r})$ to have the form of a vortex lattice, as will be elaborated below (see also SM2 [10]). Since the Abrikosov vortex lattice shares with the self consistent pair-potential in the lowest LL approximation [3] the feature of main interest here (i.e. the topological singularity of the vortex-lattice cores) this is a reasonable assumption.

A very important information is encoded in these matrix elements[1]: The zeros of the diagonal ground LL matrix-element $\Delta_{0,0}(\mathbf{k})$, are all of the first order and form a lattice dual to the Abrikosov vortex lattice rotated by 90° . Matrix elements (diagonal as well as off-diagonal) with higher LL indices have also zeros of higher orders. In the diagonal LL approximation, which is valid sufficiently close to H_{c2} , and in the case where n_F coincides with a LL index, $n = n_F$ (i.e. corresponding to particle-hole symmetry in the normal state), one trivially solves the BdG equations to find the quasi-particle energies: $\varepsilon_{\pm}(\mathbf{k}) = \pm |\Delta_{n,n}(\mathbf{k})|$, and the corresponding eigenstates: $(u_{n\pm}(\mathbf{k}), v_{n\pm}(\mathbf{k})) = (\pm e^{i\phi(\mathbf{k})/2}, e^{-i\phi(\mathbf{k})/2})/\sqrt{2}$,

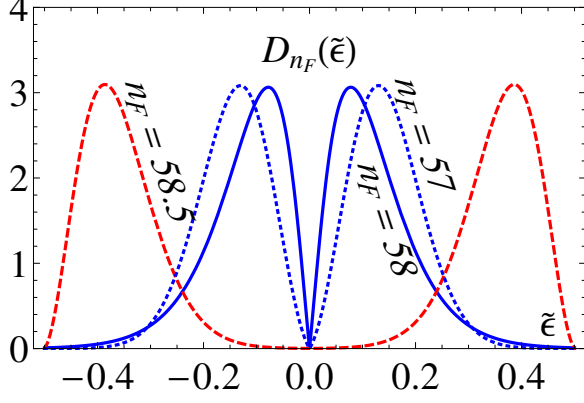


FIG. 1: Quasi-particle DOSs, as functions of $\tilde{\epsilon} \equiv \epsilon / \hbar \omega_c$, calculated by solving the BdG equations (1) near the Fermi energy ($\epsilon = 0$) for integers n_F ($= 57, 58$) and a half integer n_F ($= 58.5$) with $\Delta_0 / \hbar \omega_c = 1$. Note the sharp increase of the Dirac-shaped DOS slop from $n_F = 57$ to 58 .

with: $e^{i\phi(\mathbf{k})} = \Delta_{n,n}(\mathbf{k}) / |\Delta_{n,n}(\mathbf{k})|$. Thus, near each vortex core, \mathbf{k}_j , in the reciprocal vortex-lattice [1], where $|\Delta_{n,n}(\mathbf{k})| \rightarrow \eta_n |\mathbf{k} - \mathbf{k}_j| \rightarrow 0$, the corresponding quasi-particle dispersion relation exhibits a conical intersection of the \pm branches at the chemical potential (zero energy), in close similarity to graphene Dirac cone structure on a single honeycomb sub-lattice [11],[12]. Taking into account off-diagonal LL pairing, which is crucial at magnetic fields even slightly away from H_{c2} , requires numerical solution of Eq.(1) with a great loss of physical insights. However, the topological nature of the vortex cores singularity indicates that significant fingerprints of this singularity should appear in the dispersion relation under the influence of off-diagonal LL pairing. The results for the quasi-particle density of states (DOS), shown in Fig.(1), support this conjecture: Each broadened LL splits into two sub-bands due to Andreev scattering of quasi particle-quasi hole in the vortex regions (see e.g. [13]). The split sub-bands join to form straight-line intersection whenever the chemical potential is located in the middle of a Landau band and the filling factor of the Landau levels, $\mu / \hbar \omega_c = n_F + 1/2$, is half-integer (n_F is integer). The linear vanishing of the quasi-particle DOS at the Fermi energy (see Fig.1) is a consequence of avoiding crossings, close to conical intersections, of quasi-particle energy branches at many wave vectors in the 2D magnetic Brillouin zone (see SM 1). At non-integer n_F , where the two (particle-hole) branches of the quasi-particle energy in the diagonal approximation do not intersect, the exact energy spectrum develops a gap between the two branches, $\pm |\Delta_{n,n}(\mathbf{k})|$, with a very small number of states arising from off-diagonal LL pairing occupying the 'diagonal gap'. Fig.(1) illustrates the situation for a half integer n_F , where particle-hole symmetry is satisfied in both the normal and SC states. For

any other non-integer n_F value this symmetry is obeyed in the SC state, but not in the normal state. At integer values of n_F normal-electron Landau tubes cross the (cylindrical) Fermi surface and the oscillatory magnetization has maxima, whereas the minima of the oscillations occur at the center of the cyclotron gaps (i.e. at half integers n_F). The gaps developed in the quasi-particle spectrum in the vortex state at half integers n_F ensure that the minima of the oscillations also occur at half-integer values of n_F . The envelope of the magneto-quantum oscillations in the vortex state is therefore controlled by Dirac-shaped quasi-particle DOS at integer values of n_F .

To reveal the physical mechanism controlling the field dependence of the DOS one must complement the numerical approach of the BdG equations by resorting to an analytical method that is sensitive to quasi particle (Andreev) scattering in the vortex core regions. The exact Gorkov-Ginzburg-Landau perturbation approach developed in Ref.([14]) possesses the required sensitivity. Following Ref.([14]) the leading order term in the SC thermodynamic potential, that is sensitive to vortex structure, i.e. the quartic term, takes the form: $\Omega_4 = N (k_B T / 2) \tilde{\Delta}_0^4 \sum_{\nu=0}^{\infty} I_{4\nu}$, where N is the total number of flux lines threading the SC sample, $\tilde{\Delta}_0 = (\Delta_0 / \hbar \omega_c)$, and $I_{4\nu}$ is written as a 4D 'temporal' integral $I_{4\nu} = \prod_{j=1}^4 \int_0^{\infty} d\tau_j (\beta(\gamma) / \alpha) e^{-\varpi_{\nu} \tau_+ - i n_F \tau_-}$.

In this expression $\tau_+ = \sum_{j=1}^4 \tau_j$, $\tau_- = \sum_{j=1}^4 \varepsilon_j \tau_j$, $\varepsilon_j \equiv (-1)^{j+1}$, $\alpha = \sum_{j=1}^4 \alpha_j$, $\alpha_j \equiv 1 - e^{i \varepsilon_j \tau_j}$, $\beta(\gamma) = \sum_{\mathbf{G}} [e^{-\theta_- |\mathbf{G}|^2} / (1 + \gamma) + e^{-\theta_+ |\mathbf{G}|^2} / (1 - \gamma)] / 2$, where $\gamma = (\alpha_2 \alpha_4 - \alpha_1 \alpha_3) / \alpha$, $\theta_- = (1 - \gamma) / (1 + \gamma)$, and $\theta_+ = (1 + \gamma) / (1 - \gamma)$. In $\beta(\gamma)$ the summation is over the reciprocal vortex-lattice vectors, \mathbf{G} , measured in units of the inverse magnetic length $a_H^{-1} = \sqrt{eH / c \hbar}$, $\varpi_{\nu} \equiv \omega_{\nu} / \omega_c$, and $\omega_{\nu} = (2\nu + 1) \pi k_B T / \hbar$, $\nu = 0, \pm 1, \dots$ is the Matsubara frequency at temperature T . The amplitude of the SC order parameter, $\Delta_0^2 = S^{-1} \int d^2 \mathbf{r} |\Delta(\mathbf{r})|^2$, with $S = N \pi a_H^2$, is treated as a variational parameter for minimizing the thermodynamic potential $\Omega_{SC}(\Delta_0)$. The salient features of $I_{4\nu}$ are: (1) The simple oscillatory factor $e^{-i n_F \tau_-}$, revealing directly the Fourier transformed components with respect to the dHvA frequency $F = n_F H$. (2) The vortex lattice structure factor $\beta(\gamma)$ - an extension of the well known Abrikosov parameter to the high-field regime, which depends on the electronic 'temporal' variables, τ_j , through a single composite variable - γ , and on the vortex structure through the reciprocal vortex lattice vectors \mathbf{G} . Remarkably, approaching the points $\gamma = +1, -1$ the divergence of the structure factor reflects singular coupling of fermionic quasi particles to the vortex lattice. There are two types of singular contributions: (a) where $(1 \pm \gamma)$ vanishes in the denominators of θ_{\pm} , and (b) where $(1 \pm \gamma)$ vanishes in the numerators. Case (a) corresponds to singular contribution

from the entire vortex lattice (i.e. from the single terms with $G = 0$), whereas case (b) corresponds to contributions from the entire reciprocal vortex lattice, that is local in the direct vortex lattice. At the singular points, $\gamma = +1, -1$, the exponential factor $e^{-in_F\tau_-} \rightarrow e^{-2\pi i n n_F}$, contributing only purely harmonic terms to the SC free energy in the dHvA frequency $F = n_F H$.

Slightly away from the singular points that are local in the direct vortex lattice, i.e. corresponding to many Umklapp scattering channels, there are significant contributions to the SC free energy which deviate markedly from harmonic behavior. They originate from G-vectors satisfying: $|\mathbf{G}| \approx 2\sqrt{2n_F}$, namely having length close to the Fermi surface diameter. Furthermore, due to the incommensurability of the large circular Fermi surface with the fine polygonal vortex lattice, this *Fermi surface resonance* condition yields erratic jumps of $I_{4\nu}$ as a function of n_F . The final result for the first harmonic (i.e. $\tau_- = 2\pi$) of the thermodynamic potential, $\Omega_{SC}^{(1h)}$, modulated by umklapp scattering effects in the vortex lattice, up to fourth order in $\tilde{\Delta}_0$, takes the form: $\Omega_{SC}^{(1h)}/\Omega_N^{(1h)} \simeq 1 - (\pi^{3/2}/\sqrt{n_F}) \tilde{\Delta}_0^2 + [1 + w(n_F)] (\pi^3/2n_F) \tilde{\Delta}_0^4$, where $\Omega_N^{(1h)}$ is the corresponding normal state quantity, and $w(n_F)$, shown in Fig.(2), represents highly anharmonic effects of the umklapp scattering by the vortex lattice. The influence of vortex-lattice disorder is of importance near H_{c2} where random defects which pin flux lines, and/or SC fluctuations, introduce disorder to the vortex lattice. The structure factor, averaged over the disorder realizations in the white-noise limit, reduces to its singular value, and the SC free energy up to fourth order in $\tilde{\Delta}_0$, is purely harmonic, so that $\Omega_{SC}^{(1h)}$ is obtained with $w(n_F) \rightarrow 0$, i.e. very close to the well known Maki-Stephen expression [15, 16], as expanded to the same order in Δ_0 .

The great advantage of the perturbation approach just described is in the ability to derive analytical expressions, at least for the leading terms, with sensitivity to the Andreev scattering in the vortex core regions. On the other hand, at any order of the perturbation expansion the expected broadening effect of the LL is absent. To see whether the predicted erratic oscillation effect survives this broadening we have calculated the quasi-particle DOS at various integer values of n_F by numerically solving Eqs.(1). The quantum oscillation (QO) amplitude obtained from the resulting DOS, $\mathcal{D}_{n_F}(\varepsilon)$, by means of the expression: $D(n_F, T) \approx \left(\frac{m^*}{2\hbar^2}\right) \int d\varepsilon \frac{\mathcal{D}_{n_F}(\varepsilon) X}{\cosh^2(\varepsilon X)}$, derived in the low temperature limit, $X \equiv \frac{\hbar\omega_c}{2k_B T} \gg 1$, of the well-known formula for the thermodynamic DOS [11], is compared in Fig.(2) with the oscillatory modulation function $w(n_F)$. The good agreement between major features in the modulated envelopes of oscillations obtained in these calculations confirms the conjectured relation between the enhancement of the DOS slop at zero energy and

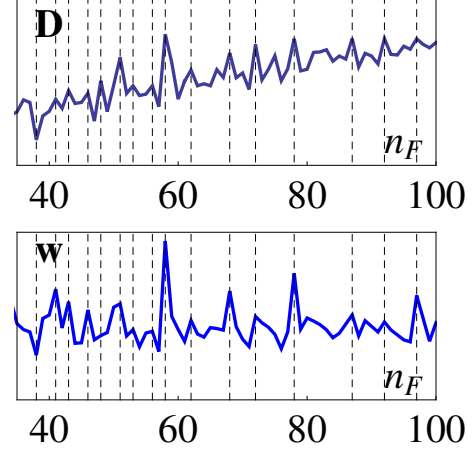


FIG. 2: Top panel: Calculated thermodynamic DOS [11], as a function of integer n_F , obtained at $\hbar\omega_c/2k_B T = 10$, from the BdG equations. Bottom panel: QO amplitude calculated at the same n_F values using perturbation GGL theory. Note the pronounced jump from $n_F = 57$ to 58 which correlates with a sharp increase of the Dirac-shape slop shown in Fig.(1).

the Fermi surface resonance-scattering through the vortex lattice cores. The most pronounced feature appearing quite similarly in both calculations is the sharp rise of the QO amplitude seen in Fig.2 in going from $n_F = 57$ to 58 , which is correlated with a sharp increase in the slop of the DOS at $\varepsilon = 0$ (see Fig.1). The corresponding dispersion relation for $n_F = 57$ (see SM1 [10]) preserves the ideal conical intersection at the vortex core which characterizes the diagonal LL branches $\pm |\Delta_{n,n}(\mathbf{k})|$. Note, however, that the Dirac-shaped DOS appearing at integer n_F values is determined by the entire landscape of avoiding crossings close to conical intersections appearing in the 2D BZ rather than by a single Dirac cone at the vortex core (see SM1 [10]).

Experimental observation of the predicted effect is expected in strongly type-II, layered superconductors in which magneto- quantum oscillations can be observed in the mixed state [3, 17]. The only relevant example known so far is $\kappa - (ET)_2 Cu(SCN)_2$, which was studied rather intensively by several groups[18],[19]. Sufficiently small interlayer transfer integral (i.e. about 0.04 meV[20]) on the scale of the cyclotron energy (which is about 0.13 meV at $H = 4$ T, see below), ensuring 2D electron dynamics, and clear dHvA oscillations in the SC mixed state with significant SC-induced extra damping, have been reported for this material. A modified version of the Maki-Stephen relaxation time approximation [15, 16], which takes into account the effect of SC fluctuations in the vortex liquid state[3] has shown a very good quantitative agreement with the observed data [3],[21]. The best fitting value of the mean-field H_{c2} ($T \rightarrow 0$) obtained in this analysis was 4.7 T, which is consistent with the H-T phase diagram derived in Ref.([19]). However,

observation of the predicted Dirac Fermions fingerprint on the dHvA oscillation depends on whether freezing of the vortex liquid occurs in the field range where superconductivity and dHvA oscillations coexist (i.e. for $H \sim 4.2 - 4.7$ T). Ignoring possible quantum fluctuation effects this can occur at sufficiently low temperatures where thermal fluctuations are suppressed. The data reported in Ref.([18]) (measured at $T = 20$ mK) seems to indicate that an ordered 2D vortex lattice indeed appears in the field range $H \approx 4.2 - 4.4$ T. To substantiate this statement we have calculated the quasi-particle DOS using the BdG equations for a 2D vortex lattice with the field-dependent order-parameter amplitude calculated in the mean-field approximation (which is a good approximation in the field range investigated [3]) and with parameters adopted from Ref.[18] and from the best fit reported in Ref.([3]) (see SM2-[10]). The thermodynamic DOS oscillation was calculated by first convoluting the quasi-particle DOSs with a Lorentzian of width γ determined by the measured Dingle temperature in the normal state to take into account the effect of atomic-lattice disorder. In the relevant field range the order-parameter amplitude is large (i.e. $\tilde{\Delta}_0 \sim 3$) so that the Landau bands fill essentially the entire cyclotron ‘gaps’ (the broad-bands region). Two pairs of DOSs calculated at $n_F = 155, 155.5$ and $n_F = 161, 161.5$, bordering the field range of interest, are shown in Fig.(3). In the effective energy interval (2γ) around the Fermi surface which dominates the QO, the DOS reduction in going from $n_F = 155$ to 155.5 is much larger than in going from 161 to 161.5 , though in both cases the bands essentially fill the entire cyclotron energy interval. Thus, in the region of broad Landau bands, i.e. for n_F values above about $n_F = 155$, the field modulation of the thermodynamic DOS is controlled by the quasi-particle scattering trajectories crossing vortex-lattice cores, rather than by the usual crossing of Landau tubes through the Fermi surface. The resulting calculated QO pattern, shown as inset in Fig.(4) as a function of $1/H$, exhibits a crossover from regular oscillations, with a monotonically decreasing amplitude, to a ‘irregularly’ modulated pattern, reflecting the sharp field-modulation of the quasi-particle DOS under vortex-lattice cores scattering. The calculation reproduces reasonably well a crossover of the same type, starting at about $H = 4.35$ T ($n_F = 156$) in the experimental data presented in Fig.(4).

Some readers might argue that the sharp features seen in the experiment are just noise. It should be stressed, however, that the magnitudes of the calculated features (see, e.g. the sharp changes of the DOS with n_F , and their fingerprints on the magneto-oscillations shown in SM2 [10] between $n_F = 155$ and 157) are seen to be comparable to those seen in the experimental data (Fig.(4)). It is therefore plausible that the experimentally observed features arise from the predicted effect.

It should be also noted that a similar calculation done

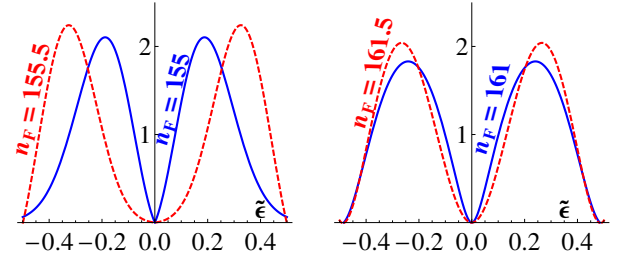


FIG. 3: DOSs, as functions of $\tilde{\varepsilon} \equiv \varepsilon/\hbar\omega_c$, calculated by solving the BdG equations 1 in the broad-band region ($\tilde{\Delta}_0 \sim 3$) at $n_F = 155$ (left panel, solid line) and 155.5 (dashed line), and at $n_F = 161$ (right panel, solid line) and 161.5 (dashed line).

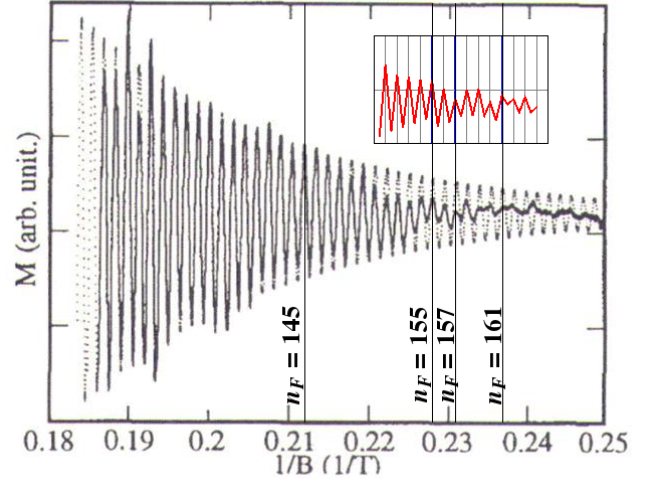


FIG. 4: dHvA oscillation above and below mean-field $H_{c2} \approx 4.7$ T ($n_F = 145$) at $T = 20$ mK reported in Ref.[18]. The dashed line extrapolates the normal state oscillation to the SC region. The inset represents calculated QO obtained by solving the BdG equations for a square vortex lattice as described in the text. Note the same scale on horizontal axes (n_F) for the main figure and the inset.

for the hexagonal vortex-lattice model (not shown) has yielded in this broad-bands region smoother features as compared to the square-lattice calculation. In the narrow Landau bands region sharp features essentially similar (but different in their details) to those shown in Figs.1,2 were obtained for the hexagonal lattice model.

In conclusion, we have revealed a fundamental relationship between Fermi surface resonance quasi-particle scatterings through vortex lattice cores and formation of pronounced Dirac Fermion structures in the reciprocal vortex lattice of a 2D strongly type-II superconductor at high magnetic fields. The predicted effect can be detected by finely tuning the external magnetic field through resonant Andreev scattering channels which sharply modulate the quasi-particle density of states. The effect is shown to leave an observable fingerprint on magneto-

quantum oscillations in the vortex-lattice state of a quasi 2D organic superconductor and could be directly detected in future scanning tunnelling spectroscopy measurements. Since vortex-lattice disorder is expected to suppress the effect [14] its observation could be used to identify the freezing transition into the vortex-lattice phase.

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Field-controlled conical intersections in the vortex lattice of quasi 2D pure strongly type-II superconductors at high magnetic fields: Supplemental material

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Supplemental material 1: Landau band-structure in the Brillouin zone and Dirac-shaped density of states at half integer filling factors

In this supplemental material we illustrate how the complex Landau band structures obtained from the BdG equations (Eq.(1) in the main text) for relatively large integer values of n_F transform into the simple Dirac-shape density of states (DOS) functions, shown e.g. in Fig.(1) of the main text. We also show here how the landscape characterizing such a band structure can be controlled by varying the Landau-level filling factor, $F/H+1/2 = n_F+1/2$, e.g. through the magnetic field intensity H . This is illustrated here for the step from $n_F = 57$ to 58 where the most pronounced jump in the quantum-oscillation amplitude is seen in Fig.(2) of the main text. Starting with a 3D plot of the Landau band-structure in the entire BZ for $n_F = 58$ shown in Fig.(1) (top-right panel) it is seen to have a very rich landscape decorated with many avoiding crossings close to conical intersections at zero energy (mostly with non-circular directrices), which reflect the influence of the vortex-lattice cores singularity. The linear energy dependence of the resulting DOS function near zero energy, shown in Fig.(1) of the main text, is determined by integrating over all these 'conical intersections'. Note, however, that at the vortex core position the band structure for $n_F = 58$ has no Dirac cone but a crater at the top of a large peak (Fig.(1) bottom-right panel). Upon varying n_F from 58 to 57 the high energy states contributing to this peak are shifted away from the vortex core, clearing the landscape for the Dirac cone at the vortex core to be visible (see Fig.(1) left panels and Fig.(2)).

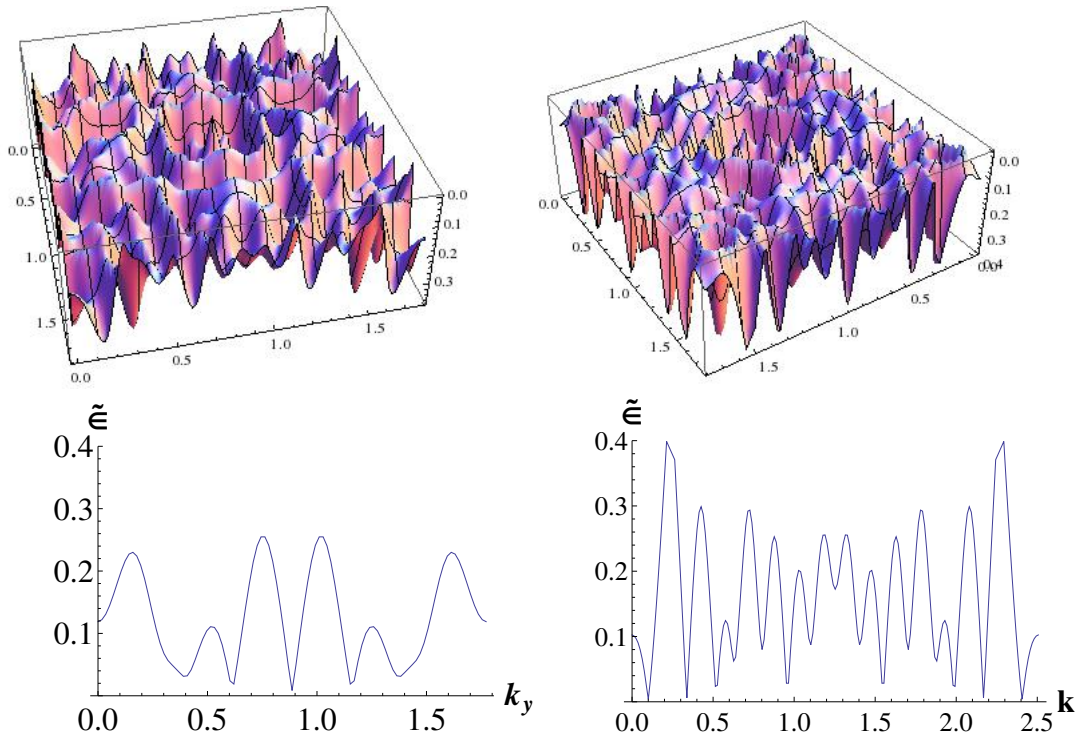


FIG. 1: Top panels: Landau band-structures for $n_F = 57$ (left) and $n_F = 58$ (right), calculated for the square Abrikosov vortex lattice in the entire 2D BZ (note the inverted energy axes). Bottom panels: 2D plots along symmetry lines passing through the vortex core: For $n_F = 57$ along the k_y -axis (left panel), whereas for $n_F = 58$ along the diagonal axis (right panel). The amplitude of the pair-potential was selected to be: $\Delta_0(H)/\hbar\omega_c = 1$.

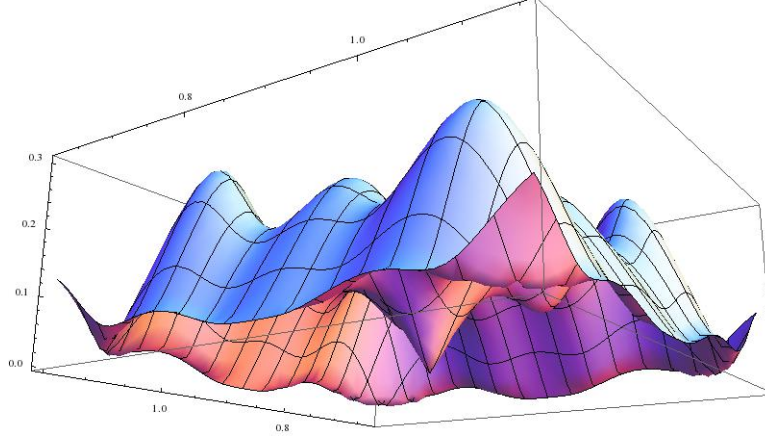


FIG. 2: The energy dispersion in a restricted range around the BZ center (reciprocal vortex-lattice core position) for $n_F = 57$, showing an extension of the central part of the top-left panel in Fig.(1) with a clear Dirac cone at the vortex core position.

Supplemental material 2: Comparison with experimental dHvA oscillations data

To compare our theory with the available low temperature (20 mK) experimental data of dHvA oscillations in the vortex state of the quasi 2D organic superconductor $\kappa - (ET)_2 Cu(SCN)_2$ [1] we use the Abrikosov lattice form for the pair potential [2]:

$$\Delta(\mathbf{r}) = \left(\frac{2\pi}{a_x^2}\right)^{1/4} \Delta_0 e^{ixy} \sum_k e^{-i\theta k^2 + iq_k x - (y+q_k/2)^2}, \quad (1)$$

$$q_k = \frac{2\pi k}{a_x}, k = 0, \pm 1, \dots, a_x^2 = \pi / \sqrt{1 - (\theta/\pi)^2}$$

where the coordinates \mathbf{r} are measured in units of the magnetic length $a_H = \sqrt{ch/eH}$ and a_x is the lattice constant along the main principal axis. In Eq.(1) $\theta = 0$ corresponds to the square lattice whereas $\theta = \pi/2$ for the triangular lattice. For the amplitude Δ_0 we use the simple mean-field (BCS) form:

$$\frac{\Delta_0(n_F)}{\hbar\omega_c} = 1.31 \left(\frac{T_c[K]}{H[T]}\right) \left(\frac{m_c^*}{m_0}\right) n_F \left(1 - \frac{F/H_{c2}}{n_F}\right) \quad (2)$$

where m_c^* is the cyclotron effective mass, m_0 the free electron mass and $F = n_F H$ is the dHvA frequency. For the detected signal in the vortex state [1] $F = 680$ T, $\left(\frac{m_c^*}{m_0}\right) = 3.5$, and mean-field H_{c2} which best fits the extra damping in the vortex liquid state is 4.7 T[2],[4].

The quasi-particle DOS, $\mathcal{D}_{n_F}(\varepsilon)$, obtained from the solutions of Eq.(1) in the main text at various values of n_F is convoluted with a Lorentzian of width $\gamma = 2k_B\pi T_D$, determined by the measured Dingle temperature T_D in the normal state, to take into account the effect of atomic-lattice disorder in the simple relaxation time approximation, i.e.:

$$\mathcal{D}_{n_F}^\gamma(\varepsilon) = \frac{1}{\pi} \int d\varepsilon' \mathcal{D}_{n_F}(\varepsilon') \frac{\tilde{\gamma}}{(\varepsilon' - \varepsilon)^2 + \tilde{\gamma}^2}, \tilde{\gamma} \equiv \frac{\gamma}{\hbar\omega_c}$$

The thermodynamic DOS (or quantum capacitance) oscillation is finally calculated by means of the expression:

$$D(n_F, T; \gamma) \approx \left(\frac{m^*}{2\hbar^2}\right) \int d\varepsilon \frac{X}{\cosh^2(\varepsilon X)} \mathcal{D}_{n_F}^\gamma(\varepsilon) \quad (3)$$

derived in the low temperature limit, $X \equiv \frac{\hbar\omega_c}{2k_B T} \gg 1$, of the well-known formula for the thermodynamic DOS [3]. The value of T_D determined from the normal state dHvA oscillation damping [5] yields $\gamma = 0.28$ K, that is $\tilde{\gamma} \approx 0.16$ at $H = 4.5$ T. Using the set of parameters, described above, in Eq.(3), the resulting QO as a function of integer and

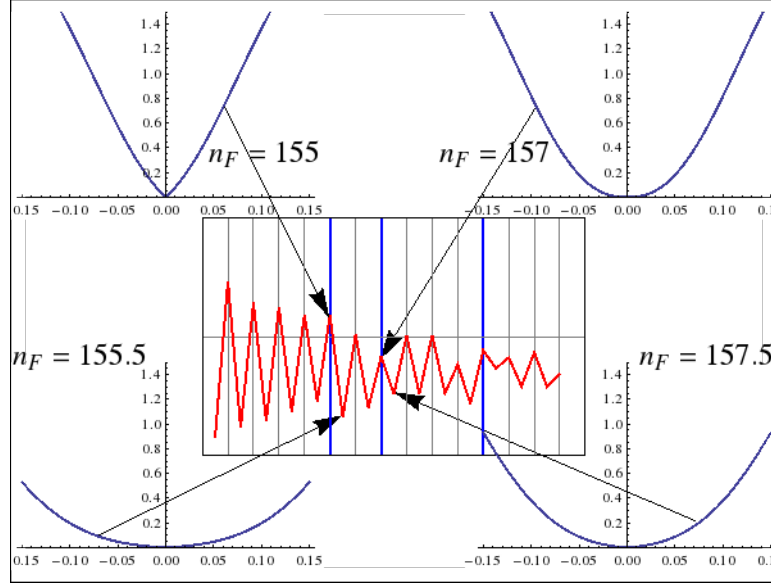


FIG. 3: Calculated $D(n_F, T; \gamma)$ in the range $n_F = 151 - 164$ ($H = 4.5 - 4.15$ T) (Central panel), showing sharp amplitude modulation between $n_F = 155$ and $n_F = 157$ associated with the suppression of the Dirac-shaped DOS shown in the upper-left panel (i.e. for $n_F = 155$) upon moving to $n_F = 157$ (upper-right panel).

half-integer values of n_F is shown in Fig.(3). DOS plots in the effective quasi-particle energy range of $-\tilde{\gamma} \lesssim \varepsilon \lesssim \tilde{\gamma}$ for $n_F = 155, 155.5$ are compared there to similar plots for $n_F = 157, 157.5$. The sharp reduction of the DOS in going from $n_F = 155$ to 157 and to 155.5 and the moderate reduction in going from $n_F = 157$ to 157.5 are the origins of the sharp drop of the oscillation amplitude seen between these points.

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